Strategic Voting and Ticket-Splitting in Mixed Electoral Systems: A Finite-Mixture Approach Applied to the Case of Germany

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Paper presented at the
The SPSA Annual Conference 2019 & Dreiländertagung
ETH Zürich
14–16 February 2019

Abstract

Why voters split their votes between elections at different (state vs federal) levels or between within dual ballots in mixed electoral systems has been subject of a debate for several years now. A natural explanation is that ticket-splitting in mixed electoral systems is the result of various voting strategies: wasted-vote avoidance in the plurality vote on the ballot, threshold insurance in the proportional vote on the ballot, or a combination of both. Uncovering such voting strategies however has posed a considerable challenge, as they cannot be observed directly.

Despite the importance of strategic voting no consensus has yet been reached on how to measure it. The proposed paper applies a finite-mixture discrete choice model to the case of ticket-splitting in the context of the mixed electoral system of Germany. Based on the German Election studies of 2009 and 2013 it derives the proportion of ticket-splitting that can be attributed to various modes of strategic voting as well as other motives

1 Introduction

In several electoral systems, voters have more than one vote that they can cast when they are called to the ballot box. In the United States on a Presidential election day, voters not only can choose which candidate they want to support in his or her bid for the US Presidency, but also which candidate they want to support in his or her bid for a seat in the House of

Representatives. Even some elections for a single political body, e.g. for the Bundestag in Germany or for the New Zealand Parliament, allow voters to cast two votes. In situations like this, voter can vote either for the same party or candidates of the same parties with both votes (a "straight ticket" vote) or can vote for different parties with their different votes (a "split ticket" vote). Various motives have been discussed in the literature as motives for split-ticket voting. In American case, voters may split their votes between their Congressional and Presidential votes when they are more centrist than any of the candidates that run for office at the different levels of government (Fiorina 1996; Bean and Wattenberg 1998). In particular with regards to ticket-splitting in mixed electoral systems with dual ballots (such as in Germany or New Zealand) such policy balancing (Kedar 2005) may also be a motive, but quite often ticket-splitting is attributed to the fact that the two votes in a dual ballot electoral system provide different incentives to vote strategically with the two votes (Schoen 1999; Pappi and Thurner 2002; Shikano et al. 2009; Gschwend 2007; Moser and Scheiner 2009).

The main challenge for testing strategic voting explanations for ticket-splitting is that it is not easy to verify whether a vote was actually cast as a strategic or non-strategic, "sincere" one, if strategic voting is understood as a vote that deviates from the voters' actual order of preference among the alternatives. Questions about voters' motives have been used as indicators for voters' decision to vote strategically only in the context of British election studies (Heath et al. 1991; Niemi et al. 1992). Most often researchers had to resort to more indirect ways to assess whether votes are strategic, whether this concerns voting behaviour in single-member district plurality systems (Alvarez and Nagler 2000), or dual ballot mixed electoral systems. For example Gschwend (2007) tries to establish the role of strategic motives in ticket-splitting via the impact that various aspects of the competitive context have on whether voters cast a straight-ticket or split-ticket vote. These approaches can make it plausible that strategic considerations are a relevant ingredient of observable rates of split-ticket voting, however they cannot provide explicit estimates of how many of split-ticket votes are strategically or non-strategically motivated.

The paper presents a novel approach at examining the role of strategic voting for ticket-splitting, based on a finite mixture discrete choice modelling approach. This approach has already been successfully applied to strategic voting in elections to the UK House of Commons (Elff 2014). In the present paper, it is shown how this model can be used to give explicit estimates of the degree to which different strategic or non-strategic motives may lead to ticket-splitting, and to what degree the amount of split-ticket votes can be traced back to various strategic or non-strategic motives. Based on such estimates, the paper also test several hypotheses about the effectiveness of the incentives that the German electoral system sets for strategic voting and for splitting one's ticket for strategic purposes.

2 The Finite Mixture Approach to Strategic Voting and other Phenomena

The point of departure of the finite mixture approach is the notion of strategic voting as deviating from one's original, "real" or "sincere" preferences in order to be more effective in bringing about a preferred outcome, more effective than by voting directly according to these sincere preferences (Fisher 2004). If a strategic vote *deviates* from a (then counterfactual) sincere vote, this leads to some considerable methodological challenges. It is usually not possible to observe whether the vote of an individual is a sincere or a strategic one, as a consequence, it is not immediately possible to determine to what degree an observed vote reflects the individual's actual preferences. In some British electoral studies, respondents were asked about the reasons of their choices and classified as sincere or strategic based these stated reasons (Heath et al. 1991). But often such stated reasons are not available and some scholars mistrust stated reasons as valid measures of strategic voting. Therefore, strategic voting has been identified with deviations from certain predictions about voting choices (**?**) or with deviations from sincere votes, which in turn are reconstructed as (counterfactual) predictions from a model of sophisticated voting where strategic considerations set to zero Alvarez and Nagler (2000). The latter two approaches do not address the fundamental dilemma created by the fact that observed choices may or may not reflect voters preferences and therefore can lead to biases and paradoxes in the estimation of strategic voting. The proposed finite mixture model addresses this dilemma by directly taking into account that an observed vote may or may not be a sincere vote, or more formally, that the observed distribution of votes is a finite mixture of several (unobserved) distributions, one of which is the distribution of sincere votes and other mixture components are guided by various motives to potentially deviate from these sincere votes.

The model distinguishes between different modes of voting, a "sincere" mode where voting choices reflect an individual's actual preference order and one or several "tactical" or "strategic" modes where an individual potentially departs from his or her preference order. Each strategic mode can be seen as connected with the desire to influence a certain aspect of the electoral outcome that may or may not be realised by a sincere vote. The classical British-style or Durvergerian strategic voting mode is based on the desire not to waste one's vote for hopeless candidates but instead to maximise one's influence on the electoral result by only voting for viable candidates that have a plausible chance to win a seat. A typical example would be a ardent Labour supporter in Colchester where only the Conservative and the Liberal Democratic candidate have a plausible choice to win the seat. This supporter might not be happy to see the Conservative candidate to win so he or she might cast his or her vote for the Liberal Democrat candidate instead of the one from the Labour party. Another possibility of strategic voting is specific to proportional representation systems that typical lead to coalition governments. A voter in Germany, who supports the CDU/CSU but realises

that no party will win an absolute majority of seats may prefer his or her supported party to form a coalition with a smaller party closer to his or her ideological stance, e.g. the FDP, than the other major contender, the SPD. In particular, if the smaller party's representation is jeopardised, because its expected vote share is near the formal threshold of representation, this CDU/CSU supporter may "lend" her vote to this smaller party. This kind of strategic voting is also known as "threshold insurance" (Gschwend 2007; Shikano et al. 2009).

In the model, each of these modes of voting are connected with a particular way in which voters restrict the set of alternatives that they consider for being chosen. The British wasted vote avoider will restrict his or her attention to only those (typically two) parties or party candidates that are viable contenders for the district seat. The threshold insurer may restrict his or her attention for the purpose of insuring the representation of a minor party to those parties that have expected vote shares near the threshold of representation. Such a consideration set typically is a proper subset of the full set of available alternatives, the choice set, unless the mode in question is that of "sincere voting". However, whether a party is in the consideration set connected to the mode a voter is currently in does not influence his or her evaluation of the parties or the utility he or she assigns to them. The fact that the Labour supporter in Colchester of the earlier example does not expect his preferred party to have any chance winning the seat does not alter his strong preference for Labour over the Conservatives.

The idea according to which strategic voting can be understood in terms of choosing from a limited consideration set differs in a crucial way from the notion of strategic voting as being guided by a utility function that involves the competitive situation, that is, in which the competitive situation effectively alters an individual's preference order over the alternatives. Utility functions that are modified by strategic considerations form the core of a popular approach at reconstructing or estimating strategic voting that goes back to Alvarez and Nagler (2000). However such utility functions may have some counter-intuitive implications: If the chance of winning is an additive component of parties' utilities, then for example the fictitious Labour supporter in Colchester may end up having a "sophisticated" preference for the Conservatives over Labour, while his actual strategic voting decision is intended to prevent the Conservatives from winning the seat in the first place.

The finite mixture model of strategic voting can be formalised as follows: Let V_i denote the (discrete valued) random variable that represents the voting choice of individual i so that if he or she voters for the alternative (party or candidate numbered) j then $V_i = j$. Let Pr() denote the probability that an event occurs, e.g. the probability that i votes for j is denoted as $\Pr(V_i = j)$. Let M_i denote the random variable that represents the mode in which individual i makes his or her choice (sincere, strategic to avoid a wasted vote etc.), so that $\Pr(M_i = h)$ denotes the probability of the event that individual i makes his or her choice in mode with number h. (In the following, $h = 1$ will usually denote a "sincere" mode of choice.) Each mode is characterised by a particular consideration set C_{hi} for each individual, so that $M_i = h$

implies $V_i \in C_{hi}$. Typically these consideration sets are proper subsets of the full choice set $\mathcal{S}_i$ of alternatives available to individual i, except for the consideration set that corresponds to a sincere vote, for which $C_{1i} = \mathcal{S}_i$. For example, the consideration set that corresponds to a strategic choice with the intention to avoid wasting one's vote contains only the candidates or parties that are viable contenders of the seat for which i casts his or her vote. The fundamental assumption then is

$$j \notin C_{hi} \Longrightarrow \Pr(V_i = j | M_i = h) = 0.$$

In the following the following abbreviations are used:

$$\pi_{ij} := \Pr(V_i = j), \quad \pi_{ij|h} := \Pr(V_i = j | M_i = h) \quad \text{and } \varphi_{hi} := \Pr(M_i = h),$$

so one can write

$$\pi_{ij} = \Pr(V_i = j) = \sum_h \Pr(V_i = j | M_i = h) \Pr(M_i = h) = \sum_h \pi_{ij|h} \varphi_{ih}.$$

For further analysis it is convenient to construct random variables that have the same distribution as the conditional distribution of V_i given $M_i = h$ by defining:

$$\Pr(U_{i|h} = j) := \Pr(V_i = j | M_i = h)$$

One could interpret $U_{i|h}$ as the potentially counter factual choice of individual i, e.g. if we somehow knew that his or her vote was strategic (which implies $M_i \neq 1$) then $U_{i|1}$ would be her choice if she had voted sincerely.

Typically, the conditional choice probabilities $\Pr(V_i = j | M_i = h)$ depend certain covariates if $j \in C_{hi}$. A parsimonious way to model the dependence of the conditional probabilities on the covariates is the conditional logit specification:

$$\pi_{ij|h} = \frac{\exp(\boldsymbol{x}'_{ij}\boldsymbol{\alpha})}{\sum_{k \in C_{hi}} \exp(\boldsymbol{x}'_{ik}\boldsymbol{\alpha})}, \text{ for any } j \in C_{hi}, \tag{1}$$

where $\boldsymbol{x}_{ij}$ is a vector of attributes of the alternative j (e.g. the evaluation of alternative j by individual i), and $\boldsymbol{\alpha}$ is a vector of coefficients.

In a similar way, a baseline multinomial logit specification can be used to model the influence of district-level or individual-level covariates on the propensity to engage in any of several modes of choice:

$$\varphi_{hi} = \frac{\exp(\boldsymbol{z}'_{h-1i}\boldsymbol{\beta}_{h-1})}{1 + \exp(\sum_g \boldsymbol{z}'_{g-1i}\boldsymbol{\beta}_{g-1})}. \tag{2}$$

for $h > 1$, where $\boldsymbol{z}_{hi}$ is a vector of district-level or individual-level covariate values and $\boldsymbol{\beta}_{h-1}$ is a coefficient vector in the logit equation for $\log \frac{\varphi_{hi}}{\varphi_{1i}}$. A special case of this specification is the

"intercept-only" variant, where $z'_{h-1i}\boldsymbol{\beta}_{h-1} = \beta_{h-1}$, which can be used if one is not interested in the influence of such covariates, but only in the relative prevalence of the modes of choice.

Based on this model, it is possible to distinguish between a vote that is cast in a strategic mode (say, $M_i = 2$) and a strategic vote. A vote may be straightforward in so far as strategic considerations lead to the same outcome as a sincere choice (under $M_i = 1$), which occurs if $U_{i|1}$ happens to be in C_{2i}, so that $V_i \in C_{2i}$ both when $M_i = 1$ and $M_i = 2$. A strategic vote, as defined in Fisher (2004) is a vote that *deviates* from a (then counterfactual) sincere vote, that is $V_i \neq U_{i|1}$, which occurs if $U_{i|1} \neq U_{i|2}$ and $M_i = 2$. The probability of such a strategic vote can be shown to be:

$$\Pr(V_i \neq U_{i|1}) = \varphi_{2i} \sum_{j \in C_{2i}} \pi_{ij|2}(1 - \pi_{ij|1}) \tag{3}$$

This what a finite mixture model would predict about whether individual i will vote strategically. Often one is instead interested in whether a vote that has been cast is a strategic one. This can be described in terms of a posterior probability of a strategic vote, given the observed vote: The (posterior) probability that a vote for alternative j is a strategic one is

$$\Pr(V_i \neq U_{i|1} | V_i = j) = \frac{\Pr(V_i = j \wedge V_i \neq U_{i|1})}{\Pr(V_i = j)} = \frac{\varphi_{2i}\pi_{ij|2}(1 - \pi_{ij|1})}{\varphi_{1i}\pi_{ij|1} + \varphi_{2i}\pi_{ij|2}} \tag{4}$$

The previous paragraphs describe only the basic model of strategic voting, where each individual casts a single vote, which may be a sincere or a strategic one. What is of interest in this paper is the application of this model to voting with dual ballots as is done in German Bundestag elections. Modelling the choices on dual ballots is more complicated because of the potential interrelations between the two votes. However it is possible to exploit some rules concerning conditional probabilities. Let V_{1i} represent individual i's first vote, V_{2i} his or her second vote, and M_i her mode of choice, then

$$\begin{aligned}
&\Pr(V_{1i} = j \wedge V_{2i} = k \wedge M_i = h) \\
&= \Pr(V_{2i} = k \wedge | V_{1i} = j \wedge M_i = h) \Pr(V_{1i} = j | M_i = h) \Pr(M_i = h) \\
&= \pi_{1ij|h}\pi_{2ik|hj}\varphi_{hi}
\end{aligned}$$

where

$$\pi_{1ij|h} := \Pr(V_{1i} = j | M_i = h), \quad \pi_{2ik|hj} := \Pr(V_{2i} = k \wedge | V_{1i} = j \wedge M_i = h),$$
$$\text{and} \quad \varphi_{ih} = \Pr(M_i = h),$$

which in turn may depend on some covariates. The marginal probabilities then are:

$$\begin{aligned}\Pr(V_{1i} = j \wedge V_{2i} = k) &= \sum_h \pi_{1ij|h}\pi_{2ik|hj}\varphi_{hi} \\ \Pr(V_{1i} = j) &= \sum_h \pi_{1ij|h}\varphi_{hi} \\ \Pr(V_{2i} = k) &= \sum_h \sum_j \pi_{2ik|hj}\pi_{1ij|h}\varphi_{hi}.\end{aligned}$$

If $\pi_{2ik|hj} = \pi_{2ik|h}$ (i.e. $\pi_{2ik|hj}$ does not vary with j) then

$$\Pr(V_{2i} = k) = \sum_h \sum_j \pi_{2ik|h}\pi_{1ij|h}\varphi_{hi} = \sum_h \pi_{2ik|h}\varphi_{hi}$$

and

$$\Pr(V_{1i} = j \wedge V_{2i} = k) = \sum_h \pi_{1ij|h}\pi_{2ik|h}\varphi_{hi}.$$

The assumption $\pi_{2ik|hj} = \pi_{2ik|h}$ can be justified by the argument that if the conditional probabilities reflect voter i's preferences over the alternatives, then these preferences should not be affected by the choices made in the first or the second vote.

Note that even if the conditional independence assumption just stated holds, the *marginal* probabilities of the first and the second choices are still interdependent, i.e. $\Pr(V_{1i} = j)\Pr(V_{2i} = k)$ still differs from $\Pr(V_{1i} = j \wedge V_{2i} = k)$:

$$\Pr(V_{1i} = j)\Pr(V_{2i} = k) = \sum_g \pi_{1ij|g}\varphi_{gi} \sum_h \pi_{2ij|h}\varphi_{hi} \neq \sum_h \pi_{1ij|h}\pi_{2ik|h}\varphi_{hi} = \Pr(V_{1i} = j \wedge V_{2i} = k)$$

except for cases where the probabilities attain degenerate values.

The probability of a split ticket thus becomes

$$\begin{aligned}\Pr(V_{i1} \neq V_{i2}) &= 1 - \Pr(V_{i1} = V_{i2}) = 1 - \sum_{j \in C_i} \Pr(V_{i1} = j \wedge V_{i2} = j) \\ &= 1 - \sum_{j \in C_i} \sum_h \pi_{1ij|h}\pi_{2ij|h}\varphi_{hi}\end{aligned} \tag{5}$$

In the analysis of ticket-splitting one is often interested to what degree ticket-splitting can be attributed to strategic considerations. One can make this question more precise by asking to what degree strategic considerations lead to ticket-splitting and to what degrees observed split ticket votes can be attributed to strategic considerations. The first question essentially asks about the conditional probability of a split ticket vote given that a voter has certain

strategic considerations (or more generally is in a particular mode of choice). In the framework established above, this conditional probability is:

$$\begin{aligned}
\Pr(V_{i1} \neq V_{i2} | M_i = h) &= 1 - \Pr(V_{i1} = V_{i2} | M_i = h) \\
&= 1 - \sum_{j \in C_i} \Pr(V_{i1} = j \wedge V_{i2} = j | M_i = h) \\
&= 1 - \sum_{j \in C_i} \Pr(V_{i1} = j \wedge V_{i2} = j | M_i = h) \\
&= 1 - \sum_{j \in C_i} \Pr(V_{i1} = j | M_i = h) \Pr(V_{i2} = j | M_i = h) \\
&= 1 - \sum_{j \in C_i} \pi_{ij|1h} \pi_{ij|2h}
\end{aligned}$$

The amount to which each combination of modes of choice in the first and the second ballot decision leads to ticket-splitting has a natural estimate in the sample average of these conditional probabilities for all individuals (indexed with i) in the sample.

The second of the above question can be answered by the posterior probability of each mode of choice given that a voter has split his or her ticket. This probability is (with $j \neq k$):

$$\begin{aligned}
\Pr(M_i = h | V_{1i} = j \wedge V_{2i} = k) &= \frac{\Pr(V_{1i} = j \wedge V_{2i} = k \wedge M_i = h)}{\Pr(V_{1i} = j \wedge V_{2i} = k)} \\
&= \frac{\Pr(V_{1i} = j \wedge V_{2i} = k \wedge M_i = h)}{\sum_g \Pr(V_{1i} = j \wedge V_{2i} = k \wedge M_i = g)} \\
&= \frac{\pi_{1ij|h} \pi_{2ik|h} \varphi_{1h}}{\sum_g \pi_{1ij|g} \pi_{2ik|g} \varphi_{1g}}
\end{aligned}$$

If these posterior probabilities are averaged over all split-ticket voters, one arrives at a natural estimate for the distribution of the causes of ticket-splitting.

With the formalism so far established one can distinguish and formally describe the, for example, the following strategic modes of voting:

- "Wasted vote avoidance": A voter who tries to avoid wasting his or her vote considers only candidates or parties with a reasonable chance to win the seat in question. These may be those two parties or candidates of parties, that are in terms of expected vote share first- or second-placed. These expectations may for example formed in an adaptive way and reflect the vote shares in the district in the previous election. In a dual vote system a voter in this mode will consider the full set of alternatives for her party list vote, but only the two best-placed parties or party candidates in the district for her district-level vote. That is, for the district-level vote the voter will choose the preferred alternative from a consideration set that contains only the two (expected) best-placed ones in the district, while for the party-list vote the voter will choose from the full set of alternatives.

- "Threshold insurance": A voter who wants to assure the representation of a minor party that is a potential coalition partner of a major party will restrict his or her attention only to those minor parties in her party-list and choose his or her preferred alternative from this restricted consideration set. For his or her district-level vote, all alternatives in the choice set will be considered.

- "Combined strategic mode": A voter can also combine strategic considerations in the district-level and the party list vote, by restricting her considerations to the two viable alternatives in the district for the district-level vote and by restricting his or her considerations for the party-list vote to the potential minor coalition partners.

In addition, the model allows also to represent the following alternative mode of voting that also affects the consideration sets of a voter:

- "Preference order voting": In an electoral system such as that in Germany, the district-level vote and the party-list vote are usually referred to as the first vote ("Erststimme") and the second vote ("Zweitstimme"). It is possible that this manner of speaking misleads voters to think that the first and the second votes are supposed to express a partial preference order of the parties, such that the voter should vote for the most preferred party with his or her "first vote" and for the "second-best" party with his or her "second vote". In this case, a voter will choose the most preferred alternative from the full choice set in her "first vote". For her "second vote" such a voter will choose the preferred alternative from among all alternatives not chosen for the "first vote", i.e. the corresponding choice set contains all alternatives except that chosen for the first vote.

The next section discusses the application of this model for the analysis of split-ticket voting in Germany.

3 Incentives for strategic voting set by the German electoral system

When voters are called to participate in an election to fill the seats in the German national parliament, the Bundestag, they can cast two votes, one to select a member of the Bundestag for the relevant voting district, the *district vote* or "first vote" (*Erststimme*) and one to influence the proportion of seats allocated to the party lists that are provided by the parties. Currently there are 299 electoral districts that fill about one half of the 598 or more seats in the Bundestag, while the remaining seats are filled by party list candidates. It should be noted that, however, the overall number of seats in the Bundestag each party surpassing a formal threshold of representation gains is primarily determined by the proportion of the list votes. Situations were a party wins more seats from district votes than it would get according to the proportion

of party list votes where handled in various ways in the past. Until quite recently, before the Bundestag election of 2013, a party that would win more seats from district votes than allocated according to party list votes would receive surplus seats, so called "overhang seats" (*Überhangmandate*) that would be added to the Bundestag. From 2013 the electoral rules were revised, because due to the peculiarities of the German electoral system, in certain situations a party could lose seats by receiving more votes (a potential situation dubbed "negative voting weights" – *negatives Stimmgewicht*). The principal modification of the electoral system in 2013 is the introduction of compensatory seats (*Ausgleichsmandate*) that are added to the Bundestag as a counterweight to surplus seats to assure that the proportion of seats reflects the proportion of party list votes. Of course, the proportionality still has limits set by the electoral formula – the Sainte-Laguë method from 2009 – and formal thresholds of representation – at least 5 percent of list votes or three electoral districts won by district votes. The consequence of surplus seats and compensatory seats is that the size of the Bundestag can vary quite considerably, from a legal minimum of 598 seats to no less than 709 in 2017 (after the 2009 election the number of seats was 622 and after the 2013 election the number of seats was 630).

Due to the main role of the proportion of party list votes and the mechanisms of surplus and compensatory seats, the German electoral system is arguably not a genuine mixed electoral system, because the PR principle dominates the allocation of seats to the Bundestag. This is particularly the case after the electoral reform of 2013 that introduced compensatory seats. While until 2009 the potential of surplus seats could create incentives for strategic voting in order to influence the result at the district level in terms of the district-level or "first votes", such incentives have almost disappeared due to the electoral reform of 2013, because any particular success of a party in terms of district-level votes will be compensated by compensatory seats.

Because of the dual nature of the district-level and party-list votes, the electoral system sets certain incentives to vote strategically and, because of that, to split one's ticket, i.e. vote for different parties at the district level and the level of party lists. The incentive to avoid wasting one's vote with the district-level vote may never have been a strong one in the German electoral system, yet a strong preference for a particular candidate for a party nevertheless may motivate one to engage in this classical "Duvergerian" strategic voting. The electoral reform of 2013 has weakened the effect of the distribution of district level votes so that one can hypothesise that:

1. the tendency to vote strategically at the district level is relatively rare (as compared to e.g. strategic voting in the UK),

2. the tendency to vote strategically at the district level has declined since the electoral reform of 2013, i.e. the proportion of district-level strategic votes is smaller in 2013 and 2017 than in 2009.

While strategic voting to influence the electoral result at the district level is a noticeable element of the public discourse of UK politics, it is less so in Germany. Instead the notion of strategic voting with the motive of threshold insurance is more common in the public debate, mainly referred to as "rental votes" (*Leihstimmen*). The liberal party of Germany, the FDP, often exercised a disproportional influence in government policies due to its (former) pivotal position as a partner for the two major parties competing for filling the position of head of government (*Bundeskanzler*). The FDP also has long been notorious for vying for threshold insuring strategic votes in its electoral campaigns (*Leihstimmenkampagnen*). However, this party strategem seems to have had quite varying success. While in 2009 the FDP gained enough votes to maintain parliamentary representation and even reached government office in a coalition with the christian democratic CDU/CSU, the FDP failed to surpass the 5 percent threshold of representation in 2013 and consequently dropped out of parliament, only to return in 2017, again with a (yet more limited) chance to gain government office (in a "Jamaica" coalition with CDU/CSU and the Greens). Due to the role that are ascribed to the *Leihstimmen* for the electoral fortunes of the FDP and other considerations one can hypothesise that:

3. the tendency to vote strategically with one's party-list vote for threshold insurance is more widespread in Germany than wasted-vote avoiding with one's district-level vote,

4. during the Bundestag elections of 2013, the proportion of strategic votes for threshold insurance was much lower than in 2009 and 2017.

4 The analysis of split ticket voting in German Bundestag elections, 2009, 2013, and 2017

The following paragraphs present an analysis of split-ticket voting in the German Bundestag elections of 2009, 2013, and 2017. The focus on these elections can be justified by their recency and the potential impact of electoral reform on the rate of strategic voting. But another motive for a focus on these election is the availability of high quality data from electoral studies surveys with a relative continuity in terms of sampling, measurement instruments, and questionnaire design: the GLES pre- and post-election cross section surveys. The *German Longitudinal Electoral Study* has emerged as one of Germany's major social science research projects (Roßteutscher et al. 2018). GLES, which started operations in 2007, comprises several survey and content analysis components, but its flagship component is a cross-section survey conducted in pre- and post-election waves on occasion of each of the Bundestag elections of 2009, 2013, and 2017. These surveys are based on a country-wide multistage probability sample of about 1000-2000 respondents in each wave. In each wave, respondents are asked about their vote intentions or, if appropriate, about their postal votes in the pre-election waves and their (recalled) votes in the post-election waves. Further, each survey wave includes

standardised instruments for the measurement of electoral choices, such as party ratings, party leader ratings, and party identification, which can be used to reconstruct or predict voters' "sincere" preference's. In addition to individual-level information, the GLES data also contains the identification numbers of the electoral districts in which the respondents cast their votes.

Using the information about which electoral districts respondents reside in and are able to vote, it is possible to reconstruct the context relevant for strategic voting at the district level. At least for the elections since 2005, the German Federal Electoral Officer (Der Bundeswahlleiter 2009, 2013, 2017) provides information about electoral results at district level recalculated for the districts relevant for the following elections. That way, it is possible to reconstruct plausible expectations voters may have about which parties' candidates are viable contenders for winning the seat in their district. It is also possible to calculate quantities that describe further aspects of the district context, such as the distance from contention of third-ranked parties or the closeness of competition between the two first-ranked parties. However, in the following these contextual properties are not considered, because only the amount of strategic voting is of interest here.

Before the discussion of strategic voting as a (potential) source of split-ticket voting, it might be reasonable first to inspect how widespread the phenomenon to be explained actually is. Figure 1 shows the amount of ticket-splitting observable in the three GLES waves. It makes clear that ticket-slitting is indeed a phenomenon to be reckoned with and is increasing in prevalence. While in 2009 only 15 percent of those respondents who reported a valid vote intention or recalled vote, reported a split ticket. This proportion has increased dramatically from 2009 to 2017. In 2013 the share of ticket-splitters in the sample is already 20 percent and in 2017 the share has increased to almost 30 percent.

The first step in the application of the finite mixture model of strategic voting is to select appropriate predictors for voters' sincere preferences. To his purpose, voters' ratings of the parties, voters' ratings of party leaders, voters' identification with one of the parties, and their preferred candidate for the chancellorship (either of the CDU or the SPD) are employed as independent variables of a conditional logit discrete choice model. For this conditional logit model, the data for each wave are brought into a "long format" or "stacked format", where each data row represents a combination of respondent and party and district-level or party-list vote. For a voter who faces 6 alternatives (*CDU*, *SPD*, *FDP*, *Grüne*, *Linke*, *AfD* – other parties are excluded due to missing values for their attributes and the rarity in which respondents in the sample choose other parties) in the 2017 Bundestag election the data set in long format contains 12 rows, 6 rows for his or her district-level vote or vote intention and 7 rows for his or her party-list vote or vote intention. A dummy variable represents the district-level or party-list vote or vote intention of a voter. For example, if a voter faced with these 7 alternatives chooses the *CDU* for his or her district-level vote and the *Linke* for his or her party-list vote, then the these two choices will be represented by 14 binary values of

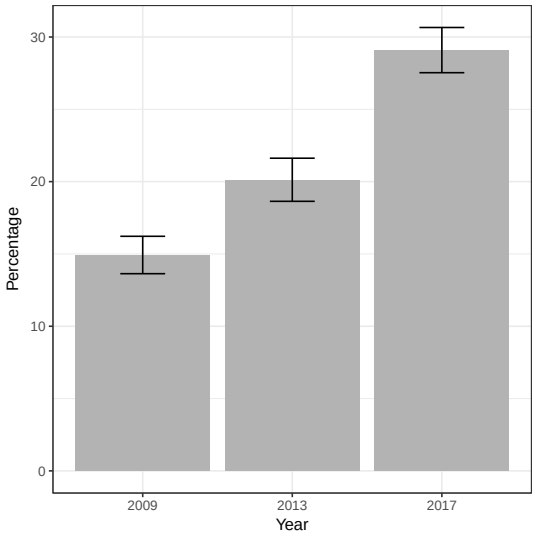

Figure 1: Split-ticket voting in the Bundestag elections of 2009, 2013, and 2017. Bars represent estimated percentages, whiskers represent 95 percent confidence intervals.

the dummy variable – the first (for the *CDU* district vote) and the 12th (for the *Linke* party-list vote) being equal to 1 and other values being equal to 0. In the survey, respondents are asked about how they rate the various parties and the ratings form the values of a single party rating variable in the long format of the data. Similarly, the party leader ratings are collected as different values of a single variable in long format, where the rating of each party leader appears in the row corresponding to each party. The rating scales are the same for all parties and party leaders. They range from -5 to $+5$ with 0 indicating a neutral rating. Party identification is represented by a dummy variable which equals 1 in those rows that correspond to the party the respondent identifies with and 0 in those rows that correspond to the other parties with which a voter does not identify. (Of course, it this variable does not attain the value 1 for any party if a respondent does not have a party identification.) Chancellorship preference is also represented by a dummy variable which is equal to 1 in the row for the CDU if a respondent prefers e.g. Angela Merkel in 2013 or equals 1 in the row for the SPD if a respondent prefers a candidate of this party (Frank-Walter Steinmeier in 2009, Peer Steinbrück in 2013, and Martin Schulz in 2017) and 0 in the other rows.

Table 1 shows the estimates and goodness-of-fit statistics of the 4 models fit to the 2009, 2013, and 2017 GLES data, where sequentially party rating, party leader rating, chancellorship preference, and party identification are entered into the set of predictors. It is obvious that none of these predictors are irrelevant, so that all four predictors are retained in the finite mixture models of strategic voting as predictors of respondents' "sincere" preferences.

Based on the chosen predictors for sincere preferences, it is now possible to specify and estimate various models of ticket-splitting and strategic voting. The models distinguish five different modes of choice in line with the previously formulated hypotheses:

Table 1: Conditional logit discrete choice models of votes and vote intentions in the Bundestag elections of 2009, 2013, and 2017

(a) Bundestag election of 2009

	(1)	(2)	(3)	(4)
Party rating	1.135***	0.892***	0.864***	0.661***
	(0.021)	(0.022)	(0.023)	(0.023)
Leader rating		0.340***	0.220***	0.185***
		(0.018)	(0.019)	(0.019)
Chancellor preference			1.003***	0.716***
			(0.048)	(0.054)
Party identification				1.260***
				(0.056)
Log-likelihood	−3413.9	−3216.4	−2993.1	−2730.8
Deviance	6827.7	6432.8	5986.2	5461.6
N	5527	5527	5527	5527

Significance: $*** \equiv p < 0.001$; $** \equiv p < 0.01$; $* \equiv p < 0.05$

(b) Bundestag election of 2013

	(1)	(2)	(3)	(4)
Party rating	1.318***	1.205***	1.081***	0.818***
	(0.024)	(0.025)	(0.025)	(0.026)
Leader rating		0.200***	0.135***	0.105***
		(0.016)	(0.016)	(0.018)
Chancellor preference			0.989***	0.771***
			(0.049)	(0.055)
Party identification				1.321***
				(0.059)
Log-likelihood	−2827.2	−2742.6	−2537.5	−2276.6
Deviance	5654.5	5485.3	5074.9	4553.2
N	5299	5299	5299	5299

Significance: $*** \equiv p < 0.001$; $** \equiv p < 0.01$; $* \equiv p < 0.05$

(c) Bundestag election of 2017

	(1)	(2)	(3)	(4)
Party rating	1.154***	0.994***	0.962***	0.703***
	(0.019)	(0.021)	(0.021)	(0.021)
Leader rating		0.219***	0.124***	0.097***
		(0.013)	(0.014)	(0.015)
Chancellor preference			0.760***	0.605***
			(0.041)	(0.045)
Party identification				1.215***
				(0.045)
Log-likelihood	−4700.7	−4558.2	−4389.3	−4005.6
Deviance	9401.4	9116.4	8778.6	8011.2
N	6172	6172	6172	6172

Significance: $*** \equiv p < 0.001$; $** \equiv p < 0.01$; $* \equiv p < 0.05$

1. a preference ranking mode: voters take the complete choice set into consideration for their "first vote" and consider for their "second vote" only those alternatives not chosen as "first vote";

2. an unrestricted or "sincere" mode, where voters take the full choice set of alternatives into consideration for their "first vote", i.e. district-level vote and their "second vote", i.e. party-list vote,

3. a wasted-vote avoiding mode: voters take only the two parties/party candidates into consideration for being chosen by their district-level vote, that were first- or second-placed in terms of vote share in the district in the previous election; for the party-list vote, voters take the full choice set into consideration;

4. a threshold insurance mode: voters take only those minor parties into consideration for the party-list vote that are regarded as potential coalition partners of either the CDU/CSU or the SPD; for the district-level vote, voters take the full choice set into consideration;

5. a combined strategic voting mode: in their first vote, voters try to avoid wasting it to candidates hopeless at district level, in their second vote, they focus on potential coalition partners.

The following models are constructed and fitted to the GLES data of 2009, 2013, and 2017:

M0: This model allows only for two modes, which are both non-strategic: a sincere voting mode and a mode where first and second votes reflect first and second preferences;

M1: this model extends M0 by a wasted-vote avoiding mode;

M2: this model extends M1 by a threshold insurance mode;

M3: this model extends M2 further by a combined strategic mode.

For each of the GLES study years, the four models are compared using likelihood ratio tests in Table 2. The test results indicate that, for a model to adequately represent the data, the inclusion of both wasted vote avoidance and threshold insurance are indispensable, whereas the inclusion of a combined strategic voting mode is not: For each of the GLES waves of 2009, 2013, and 2017, the likelihood ratio tests of M1 vs M0 and M2 vs M1 give statistically significant results against the respective null hypotheses.

The estimates of the logit coefficients of the choice modes in Model M2 for the three election years, which relate to equation (2), are shown in Table 3. Not shown are the estimates of the coefficients in the conditional choice component of the model, corresponding to equation (1), because they hardly differ from those reported in Table 1. The model does not contain any covariates for the modes of choice, thus the logit equation contains only

Table 2: Likelihood ratio tests of models with different sets of voting modes. Model M0 allows only for sincere voting and preference-order voting modes, M1 adds a wasted vote avoiding mode, M2, adds a threshold insurance voting mode, and finally M3 adds a combined strategic voting mode.

Year	Model	Deviance	χ^2	df	p-value
2009	M0	7943.8			
	M1	7881.8	62.0	1	0.000
	M2	7811.8	70.1	1	0.000
	M3	7811.8	−0.0	1	
2013	M0	4545.6			
	M1	4493.9	51.7	1	0.000
	M2	4481.1	12.8	1	0.000
	M3	4481.1	0.0	1	0.990
2009	M0	5488.8			
	M1	5444.3	44.5	1	0.000
	M2	5428.4	15.9	1	0.000
	M3	5428.4	0.0	1	0.975

intercepts, which express the relative sizes of the proportions of respondents voting in the respective choices. Since the baseline of the logit equation is the mode of sincere voting, the negative estimates indicate that the other modes are less prevalent than this baseline. It is not easy to assess the prevalence of the voting modes based on these log-odds, but they are not the quantities of interest anyway. Instead, in the following the derived quantities discussed earlier are presented in the following paragraphs. Firstly, the estimated percentages of voters using an unrestricted (sincere) mode of choice, a strategic mode avoiding wasted votes, a strategic mode intent on threshold insurance, and finally a preference order voting mode are shown in Figure 2.

Figure 2 makes clear that strategic voters are a minority among all voters. Only about 5 percent of the respondents that report valid vote decisions vote in a way consistent with the motive to avoid wasting one's vote and less then 1 percent of respondents vote in a way consistent with the motive of threshold insurance in favour of potential minor coalition partners. The proportion of voters who actually deviate from their sincere preferences in line with these motives may even be smaller. The dominant group of voters is that of those who do choose from the full set of alternatives both in their district-level and party-list vote. The next largest group are those who employ a strategic voting mode for threshold insurance. However, this group is already very small. The smallest group of voters, however, is that of those who cast their two votes coherent with their preference order among parties, i.e. their first preference with the "first vote", and their second preference with their "second vote". Apparently the overwhelming majority of German voters understand the electoral system

Table 3: Estimates for the parameters of the distribution of choice modes in the Bundestag elections of 2009, 2013, and 2017. The "sincere" or unrestricted mode is the baseline category for the logit equations.

(a) Bundestag election of 2009

	Preference order	Avoid wasted v.	Thresh. insurance
(Intercept)	-6.170^{***}	-2.906^{***}	-4.569^{***}
	(1.000)	(0.243)	(0.450)

Significance: $*** \equiv p < 0.001$; $** \equiv p < 0.01$; $* \equiv p < 0.05$

(b) Bundestag election of 2013

	Preference order	Avoid wasted v.	Thresh. insurance
(Intercept)	-5.969^{*}	-2.535^{***}	-4.338^{***}
	(2.785)	(0.230)	(0.468)

Significance: $*** \equiv p < 0.001$; $** \equiv p < 0.01$; $* \equiv p < 0.05$

(c) Bundestag election of 2017

	Preference order	Avoid wasted v.	Thresh. insurance
(Intercept)	-5.228^{***}	-2.459^{***}	-5.860^{***}
	(1.014)	(0.187)	(1.393)

Significance: $*** \equiv p < 0.001$; $** \equiv p < 0.01$; $* \equiv p < 0.05$

well enough not to be mislead by a distinction between first and second vote into thinking they are supposed to enable voters to express a first and second preference.

The amount and development of strategic voting is not fully in line with the expectations from the first two of the hypotheses formulated earlier. While the prevalence of the strategic motive to avoid wasting one's vote appears smaller than the author's finding for the UK, in consistence with the first hypothesis, the change in the electoral system introduced in 2013 does not appear to have had an impact on the prevalence of this motive. While the electoral system change almost completely eliminated the incentive to vote strategically at the district level, the proportion of voters with the motive to avoid wasting their district-level vote appears to have remained stable or even increased. The second two hypotheses do not fare better and are even completely refuted. According to the third hypothesis it is to be expected that the threshold insuring motive should be more widespread than wasted vote avoidance, due to the dominant proportional representation characteristic of the German electoral system. However this motive is less common than the other strategic motive. Furthermore, threshold insurance was not less common in 2013, as expected by the fourth hypotheses, instead the prevalence of this motive is so low that any changes from one election to the next are swamped by sampling error.[1]

[1] One could be tempted to speculate that these disappointing results are the consequence of disallowing the combination of wasted-vote avoidance in the first vote and threshold-insurance in the second vote.

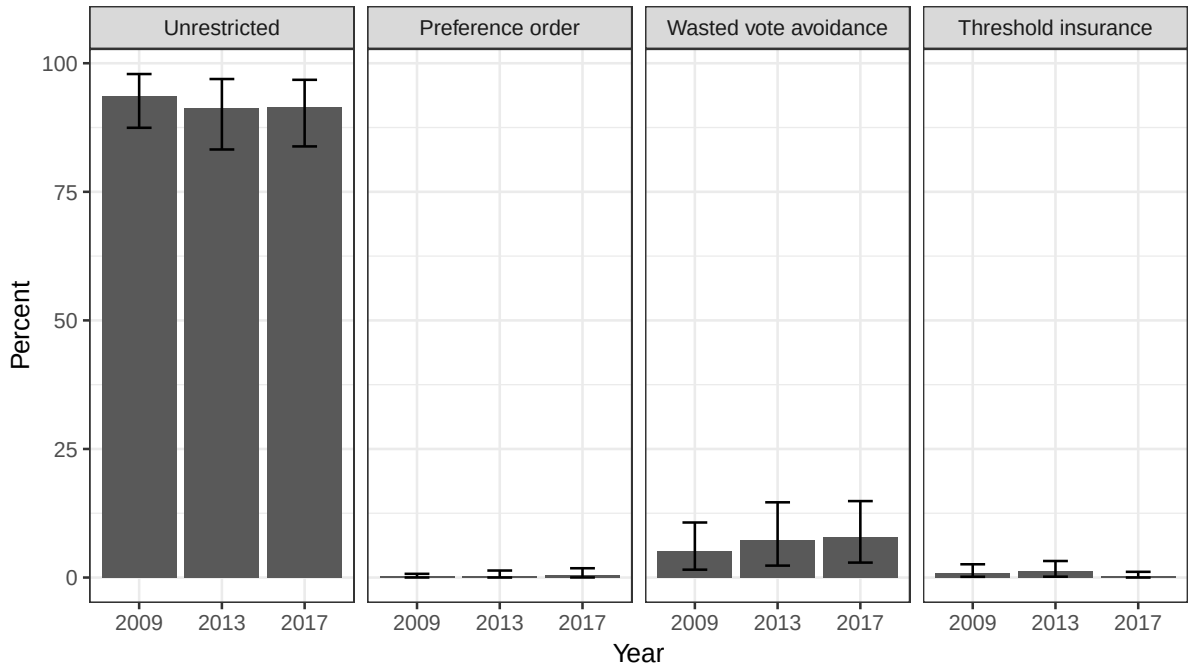

Figure 2: Estimated shares of voters in various modes of choice in the GLES samples of 2009, 2013, and 2017. Bars represent estimated percentages, whiskers represent 95 percent confidence intervals.

Figure 3 shows the distribution of the four voting modes among the ticket-splitters. It thus describes to what degree existing rates of ticket-splitting can be attributed to strategic voting or other motives. The distribution of the for modes among the ticket-splitters is similar to the distribution among all voters in the sample: With the unrestricted mode and the wasted-vote avoiding mode the most common ones, and preference order voting the least common one, with exception of 2009. Overall, strategic voting modes are more common among ticket-splitters than among all voters. That notwithstanding, Figure 3 gives little support to the hypotheses formulated earlier: Wasted vote avoidance is much more common among ticket-splitters than threshold insurance, despite the reasoning that wasted-vote avoidance should be rather uncommon in an electoral system dominated by the proportional representation principle. Further, there does not seem to exist a connection between the electoral success of the FDP and the amount of threshold insurance. Ironically, in 2013, when the FDP failed to gain representation in the Bundestag, the mount of threshold insurance voting was the highest. Maybe the prospect of the FDP failing to win representation in the Bundestag led more voters to split their tickets with the intention to increase their chances.

However, this combination can be expected to be rare: It arises only if somebody prefers one of the two major party groups SPD or CDU/CSU (a necessary condition for threshold insuring strategic voting), but finds this party to be non-viable in the district he or she votes (a necessary condition for wasted vote avoiding strategic voting).

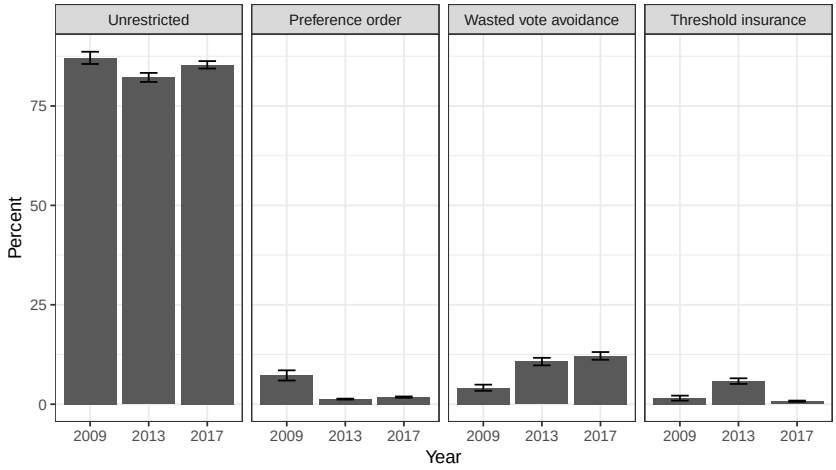

Figure 3: The distribution of voting modes among ticket-splitters. The bars represent average posterior probabilities. The whiskers represent 95 percent bootstrap confidence intervals. The proportion of voters in rigid straight-ticket mode is not shown, because it is zero among the ticket-splitters by construction.

5 Conclusion

The present paper develops a finite mixture approach to modelling strategic voting and applies this model to the study strategic voting. A crucial advantage of this approach to strategic voting is that it allows to specify explicitly several variants of strategic voting and to estimate their prevalence, without the need to make assumptions about the impact of predictors such as the closeness of competition between viable contenders or the distance from contention of nonviable contenders. When applied to split-ticket voting, the finite mixture model allows to specify and estimate the prevalence of various strategic and non-strategic motives of ticket-splitting, instead of having to infer the role of strategic motives indirectly from the (estimated) influence of certain predictors. While the influence of predictors can only make it *plausible that* strategic motives may play a role in ticket-splitting, the proposed method allows to quantify *how much* of ticket-splitting can be traced back to strategic and non-strategic voting. As demonstrated in this paper, the finite mixture model can also be used to exhibit another motive behind split-ticket voting, voting according to a preference order.

The finite mixture model was applied to the case of split-ticket voting in Germany and to test some hypotheses about the role of strategic voting as a source of ticket-splitting, namely that (1) wasted-vote avoidance should be less common in Germany than in the UK, (2) it should be decline after the electoral reform of 2013, (3) threshold insurance should be more common than wasted-vote avoidance, and (4) threshold insurance should be particularly low in 2013, when the FDP who often campaigned for "rental votes" failed to gain representation in the Bundestag. The results did not confirm these hypotheses, instead wasted-vote was more common than threshold insurance and also remained more or less a the same level before and after 2013. Threshold insurance did not have a low point in 2013, instead it was highest in

that election. Among the ticket-splitters the group of votes who use this pattern of voting to express the rank order of their preferences is almost as large as the group of those who split their ticket to avoid wasting their district-level vote.

A limitation of the paper so far is that it has not yet been worked out to what degree voting in a wasted vote avoidance mode or a threshold insurance mode actually led to strategic voting in the full sense of deviating from one's preferences. Further, the predictions about the amount of ticket-splitting conditional on the various voting types are probabilistic and model-based and do not yet take into account actual levels of split-ticket voting. This could be improved by using posterior probabilities, which have yet to be derived. Finally, there may be other types of strategic voting at work in Germany that are not so much targeted at the consequences of the electoral system than at the features of government formation after the parliament has been elected: If the motive of strategic voting is to maximise one's impact on the eventual government, then one could for example consider a vote wasted if it does not affect the composition of a government coalition.

Appendix – Model Estimation

In the following the construction of the likelihood function is discussed, which is the base for the computation of ML estimates for the finite mixture model if ticket splitting discussed in the paper. The crucial assumption of this construction is that, conditional on the mode of voting and the choice predictors (the attributes of the alternatives, such as the evaluation of the respective parties and their leaders), the probability that a voter chooses with his or her first vote is conditionally independent from his or her second vote and vice versa. At first glance this seems a quite strong assumption, but it merely means that, whatever could lead to a systematic relation between the first an the second vote is already "captured" by the choice predictors, with represent the "true" preference order of the voters. Based on this conditional independence assumption, the probability that voter i votes for alternative j_1 with his or her first vote and for alternative j_2 with his or her second vote can be expressed as:

$$\begin{aligned}\Pr(V_{i1} = j_1, V_{i2} = j_2) &= \sum_{h=1}^{q} \Pr(V_{i1} = j_1 | T_i = h) \Pr(V_{i2} = j_2 | T_i = h) \Pr(T_i = h) \\ &= \sum_{h=1}^{q} \varphi_{hi} \pi_{ij_1 1|h} \pi_{ij_2 2|h}\end{aligned}$$

Define dummy variables Y_{ijk} (for $k = 1, 2$) such that

$$Y_{ijk} = \begin{cases} 1 & \text{if } V_{ik} = j \\ 0 & \text{if } V_{ik} \neq j \end{cases}$$

The likelihood of the observation $\boldsymbol{v}_i = (j_1, j_2)$ given model parameters $\boldsymbol{\theta}$ (which include the $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ parameters) then can be written as a function of dummy variable values:

$$\mathcal{L}_i = \sum_{h=1}^{q} \varphi_{hi} \pi_{ij_1 1|h} \pi_{ij_2 2|h} = \sum_{h=1}^{q} \varphi_{hi} \prod_{k_1 \in \mathcal{S}_i} \pi_{ik_1 1|h}^{y_{ik_1 1}} \prod_{k_2 \in \mathcal{S}_i} \pi_{ik_2 2|h}^{y_{ik_2 2}},$$

given that $y_{ij_1 1} = 1$ and $y_{ij_2 2} = 1$.

The log-likelihood for the entire observed data then takes the form

$$\ell = \sum_i \ln \mathcal{L}_i = \sum_i \ln \sum_{h=1}^{q} \exp\left(\ln \varphi_{hi} + \sum_{j \in C_{i1|h}} y_{ij1} \ln \pi_{ij1|h} + \sum_{k \in C_{i2|h}} y_{ik2} \ln \pi_{ik2|h} \right)$$

which is only slightly more complicated than the likelihood function developed in Elff (2014). With an moderate modification of the software written to estimate the models in Elff (2014), estimates for the model developed in this paper can be computed, again with an EM algorithm (Dempster et al. 1977; Louis 1982; McLachlan and Krishnan 2007).

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